Azimuth & Elevation Estimation using Acoustic Array

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Abstract – *Tracking of helicopters or any airborne targets requires estimation of both azimuth and elevation angles of the target. In this paper, an algorithm for estimation of azimuth and elevation using acoustic array (array of microphones) will be presented. There are several super resolution algorithms for estimation of direction of arrival (DoA) angles, such as minimum variance distortionless response (MVDR), multiple signal classification (MUSIC), ESPRIT, etc. While these algorithms are able to provide fairly good estimation of azimuth, they are unable to estimate the elevation angles with reasonable accuracy. In this paper, a new pardigm is developed to estimate the pointing vector (azimuth & elevation) to the target using the angle of arrivals (AoA) of a target's signal at each pair of microphones in the array. Mathematical formulation of the problem is presented. The algorithm for estimation of azimuth and elevation is used on actual helicopter data, and the results are presented.*

Keywords: Tracking, azimuth & elevation estimation, MUSIC, MVDR, pointing vector.

1 Introduction

Acoustic sensors are increasingly playing important roles in the modern warfare. One of the advantages of the acoustic sensors is that they are omni-directional and do not require the line of sight for detection. Another advantage is that the acoustic sensors are extremely cost-effective compared to the more expensive and power-consuming traditional sensors such as RADAR, video, etc. Some of the initial applications of acoustic sensors are in acoustic transient event detections, namely, sniper detection, mortar/rocket launchings, and detonations. There are several commercial systems available for sniper localization using acoustic sensors, for example, Bomerang [1], Shotspotter [4], Pilar [3], etc. Acoustic sensors are also used for tracking and classification of both military and civilian vehicles [5], [6], [7]. Other applications for acoustic sensors are in tracking, where several acoustic sensor arrays are used to estimate the bearing angles of the targets at each sensor array, which triangulates the position of the target. Yet other applications of the acoustic ssnsors are in unattended ground sensor (UGS) applications. The majority of the UGS use acoustic sensors as activity detectors to detect any activity in the neighborhood and then wake up more expensive and/or more power-consuming sensors such as video, radar, etc. UGS are also used in intruder detection in perimeter protection applications. In this paper, the acoustic sensor arrays are used to track the helicopters. In order to track the helicopter, one needs to estimate both azimuth and elevation. There are algorithms for estimation of both azimuth and elevation using a pair of identical sensor arrays that are placed within close proximity of each other [10], and these algorithms are applied on simulated data only. Other algorithms [14, 15] on estimation of both azimuth and elevation in literature also show results on simulated data. While estimation of the azimuth is well established, elevation estimation using a single array is found to be difficult [11].

For tracking, several acoustic sensor arrays are deployed in an area. Each sensor array consists of three or more microphones in order to have 360^o coverage of the targets. The microphones in the array are arranged according to some geometrical structure, for example, in circular or tetrahedral array. Figure 1 shows tetrahedral arrangement of microphones. The microphones in Figure 1 are covered with foam balls to reduce the wind-noise. The size of the array depends on the aperture one would like to have, which in turn relates to the angular resolution. Typically, each microphone is placed at a radial distance of 1 m from the center of the array. Once the arrays are placed in the field at known coordinates, the acoustic data is collected and analyzed for the presence of targets, such as motor vehicles, helicopters, airplanes, mortar firing, and gunfire. The captured acoustic data is processed for the bearing or direction of arrival (DoA) angles of these

Figure 1: Tetrahedral acoustic array

Figure 2: Acoustic sensor array with plane wave input

targets. Once the bearing angles are obtained, the location of the target is determined using triangulation. In order to estimate the DoA angles of the targes, one of the super resolution techniques, such as minimum variance distortionless response (MVDR) [8], multiple signal classification (MUSIC), or Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [9], is used. Although these techniques provide accurate results for azimuth, they fail to provide elevation estimates. Figure 3 shows the output of MVDR algorithm on real helicopter data. It is clear from Figure 3 that the elevation angles are far from the actual ground truth.

In this paper, an algorithm for estimation of both azimuth and elevation angles is provided. Section 2 provides the basic concepts and theory. Section 3 provides the helicopter data collection and algorithm development. Section 4 presents the results of the algorithm for helicopter data. Concluding remarks are presented in section 5.

2 Theoretical Background

Traditional DOA estimation algorithms, such as MVDR and MUSIC, use all the microphones in an array for estimation of both azimuth and elevation. A brief review of the MVDR is presented here for the sake of completeness. Figure 2 shows an array of microphones located at coordinates denoted by $p_i, \quad i \in \{1, \cdots, 4\}$. The input to the array is a plane wave in the direction a with radian frequency w, where a can be expressed as

$$a(\theta,\ \phi) = [cos\theta\ cos\phi,\ cos\theta\ sin\phi,\ sin\theta]^T, \qquad (1)$$

where ϕ denotes the azimuth, θ denotes the elevation, and 'T' denotes the transpose. The received signal at each microphone is denoted by s_i and is given by

$$s_i(t) = r_i(t) + n(t),\ \forall i \in \{1, \cdots, 4\}, \qquad (2)$$

where r_i is the actual signal at the microphone due to the target and 'n' is the noise. Define the steering vector V as

$$V = [e^{-jw\frac{p_1 \bullet a}{c}}, e^{-jw\frac{p_2 \bullet a}{c}}, e^{-jw\frac{p_3 \bullet a}{c}}, e^{-jw\frac{p_4 \bullet a}{c}}]^T, \qquad (3)$$

where '$\bullet$' the dot product, 'c' is the propagation velocity of sound, and a is the direction of the plane wave impinging on the sensors. The Fourier transformation of the signal $s_i,\ \forall i \in \{1, \cdots, 4\}$ is denoted by S, and the covariance matrix at a given frequency is denoted by R. The beamformer output is given by

$$y(t) = W^H s(t), \qquad (4)$$

where W is the weight vector and 'H' denotes the Hermitian transpose. The MVDR beamformer minimizes the array output power while keeping the unit gain in the direction of the desired signal

$$\min_W W^H R W \qquad subject\ to\ \ W^H V = 1 \qquad (5)$$

A closed form solution to (5) is given [12] by

$$W_{MVDR}(w, \theta, \phi) = \frac{R^{-1}V}{V^H R^{-1} V}, \qquad (6)$$

where V depends on the variables w, θ and ϕ. Use of MVDR to estimate both azimuth and elevation did not provide good estimation of elevation angles, although it provided good azimuth estimation (see Figure 3). So, estimation of elevation angle is the main focus of the algorithm developed here. The algorithm developed here uses MVDR for inital estimation of azimuth.

In order to estimate elevation, the following approach is developed. Consider an array consisting of only two microphones, as shown in Figure 4. At each pair of microphones, the angle of arrival (AoA) can be estimated using the geometry shown in Figure 4 which is given by

Figure 3: Estimation of Azimuth and Elevation using MVDR

Figure 4: Geometry for estimation of AoA

$$\gamma = acos\left(\frac{x}{d}\right), \tag{7}$$

where 'd' is the distance between two microphones, 'x' is the additional distance the plane wave needs to travel to reach the microphone at location p_1, and γ is the AoA of the plane wave with respect to the axis joining the two microphones. The distance, x, can be determined by measuring the time difference between the signals arriving at each microphone. The time difference can be estimated either by cross correlation or by determining the phase difference between the two signals. Clearly, the AoA γ defines a cone around the axis joining the two microphones, as shown in Figure 5(a). And the equation of the cone is given by

$$\frac{(p_1 - p_2)^T \bullet U}{\|p_1 - p_2\|} = \cos\gamma,$$

where $U = a$ is the unit vector in the direction of target given by (1). Now, each AoA for each

Figure 5: (a) Single cone (b) intersection of cones

pair of microphones generates a cone around their respective axes and they intersect. Figure 5(b) shows the intersection of three cones. These cones intersect along a line that points to the target – that is, the line of intersection gives the pointing vector U to the target.

Let $Z = [0\ 0\ 0]^T$ be the center of the array, and $S_{i,j}$ is a point in the space such that the vector $S_{i,j} - Z$ is parallel to the vector $p_i - p_j$ and the magnitude of the vector $\|S_{i,j} - Z\|$ equals to $\|p_i - p_j\|$. For a small array and large target distances, the AoA of the target with respect to the vector $S_{i,j} - Z$ would be same as the AoA with respect to the vector $p_i - p_j$. Then the equation of the cone with respect to the axis $(S_{i,j} - Z)$ is given by

$$\frac{(S_{i,j} - Z)^T \bullet U}{\|S_{i,j} - Z\|} = cos\gamma_{i,j}. \tag{8}$$

Then solving for the pointing vector U implies solving the linear equation:

$$\begin{bmatrix} (S_{1,2} - Z)^T \\ (S_{1,3} - Z)^T \\ \vdots \\ (S_{3,4} - Z)^T \end{bmatrix} U = \begin{bmatrix} \|S_{1,2} - Z\| \cos\gamma_{1,2} \\ \|S_{1,3} - Z\| \cos\gamma_{1,3} \\ \vdots \\ \|S_{3,4} - Z\| \cos\gamma_{3,4} \end{bmatrix} \tag{9}$$

From U, one can easily determine both azimuth ϕ and elevation θ angles from (1). The algorithm developed here involves solving (9) for U using the ground truth of the sensor postions p_i, $\forall i$, and estimation of AoA $\gamma_{i,j}$ for all pairs of microphones.

2.1 Estimation of AoA

Let $s_i(t) = e^{-j\,wt_i}$ be the complex signal at i^{th} microphone. Then the phase difference between the two signals received at microphones i and j is

$$\psi_{i,j} = angle\left(e^{-jw(t_i - t_j)}\right)$$

Angle ψ can be estimated from the fast Fourier transform (FFT) of the data. In order to compute the AoA using (7), one needs to estimate the distance, x, or the time taken by the wave to travel the distance x, and the distance d is the distance between two microphones. The time t_{ij} taken for the wave to travel x is related the phase angle ψ. Let λ be the wave length of the signal, then

$$x = \frac{2\pi}{\psi_{i,j}}\lambda \qquad (10)$$

and

$$t_{ij} = \frac{x}{c}, \qquad (11)$$

where 'c' is the propagation velocity of sound. The time t it takes to travel the distance d is d/c. Then the AoA is given by

$$\gamma_{i,j} = acos\left(\frac{x}{d}\right) = acos\left(\frac{t_{ij}}{t}\right) \qquad (12)$$

If $t < t_{i,j}$, then $t_{i,j} = mod(t_{i,j}, t)$.

2.2 Refraction of Sound Waves

Estimation of the elevation angle is further complicated due to the propagation dynamics of the sound waves in the propagation media – air. The temperature at the ground is higher than the temperature at high altitude near the helicopter. As a result the propagation velocity of sound increases from high altitude to the ground. This, in turn, results in refraction of the sound wave [13], as shown in Figure 6. It is clear from Figure 6 that the elevation angle decreases as the sound wave propagates from the helicopter to the sensor array on the ground. This is particularly true for the data collected in a desert environment, which is the case for the data used for this paper.

Figure 6: Refraction of Sound Wave

In the next section, an algorithm for estimation of both azimuth and elevation is presented.

3 Data collection & Algorithm Development

For the purpose of collection of helicopter data, two tetrahedral acoustic sensor arrays are deployed over an area, as shown in Figure 7. A helicopter is flown in the vicinity at a range starting from 0.2 Km to 7 Km. The ground trace of the helicopter path is shown in Figure 7. The data is collected at a sampling rate of 1000 samples per second. Figure 8 shows typical helicopter data and its FFT. All the helicopters have main rotor and a secondary rotor. The main rotor rotates at a fixed speed, that is, revolutions per minute (RPM). This RPM results in a fundamental frequency of 10 – 14 Hz, which can be seen in the FFT data shown in Figure 8 and the spectrogram of the data shown in Figure 9. In general, the acoustic data generated by the helicopter main rotor blades is impulsive in nature. This generates several harmonics of the fundamental frequency that can be seen in Figure 9. Since these frequencies are effected differently due to propagation through air, it is important to use several harmonics to estimate AoAs.

Figure 7: Sensor location and the ground trace of helicopter

In general, the AoA γ could be obtuse or acute, depending on the helicopter position. Since one does not know apriori the location of the helicopter, it is necessary to estimate the AoA, assuming the helicopter could be anywhere. This is done by using one of the microphones as reference for one angle and another microphone as the reference for the second angle; in other

Figure 8: (a) one second data (b) FFT of the data

Figure 9: Spectrogram of the helicopter data

words:

$$\begin{aligned} \psi_{i,j}^{(1)} &= angle\left(\frac{s_i(t)}{s_j(t)}\right) = angle\left(e^{-jw(t_i - t_j)}\right) \\ \psi_{i,j}^{(2)} &= \pi - angle\left(\frac{s_j(t)}{s_i(t)}\right) = \pi - angle\left(e^{-jw(t_j - t_i)}\right) \end{aligned} \tag{13}$$

These angles $\psi_{i,j}^{(1)}$ and $\psi_{i,j}^{(2)}$ are used in (10) and then $t_{i,j}$ and $t_{j,i}$ will be estimated using (11), which then can be used to estimate the AoAs $\gamma_{i,j}^{(1)}$ and $\gamma_{i,j}^{(2)}$ using (12), respectively. Let $\Gamma_{i,j}^{h_q} = \left\{\gamma_{i,j}^{(1)}, \gamma_{i,j}^{(2)}\right\}$ be the set of AoAs estimated at the harminic frequency $h_q \in H$ and $H = \{h_1, \cdots, h_m\}$, where m denotes the number of significant harmonics estimated from the data and $\Gamma_{i,j} = \cup \left\{\Gamma_{i,j}^{h_q}\right\}, \ \forall q \in \{1, \cdots, m\}$.

3.1 Selection of AoAs and Estimation of ϕ and θ

The previous section presented a method for generating two AoAs $\Gamma_{i,j}^{h_q} = \left\{\gamma_{i,j}^{(1)}, \gamma_{i,j}^{(2)}\right\}$ at each harmonic frequency h_q. However, in order to estimate the azimuth and elevation of the helicopter, it is necessary to use only one AoA for each pair of microphones. This requires some guidance in selecting the proper AoA among many for each pair of microphones. In order to select the right AoA, an array of theoretical AoAs for a given azimuth ϕ and elevations $\theta \in \{0, \cdots, 90\}$ will be generated using (8). Let us denote this set of AoA by $A_{i,j} = \{\hat{\gamma}_{i,j}(\phi, 0), \hat{\gamma}_{i,j}(\phi, 1), \cdots, \hat{\gamma}_{i,j}(\phi, 90)\}$. Let $\hat{\phi}_k$ and $\hat{\theta}_k$ denote the estimated values of azimuth and elevation at the k^{th} time step; then $\hat{\gamma}_{i,j}(\hat{\phi}_k, \hat{\theta}_k)$ denotes the expected AoA.

Estimation of azimuth and elevation is done as follows:

Azimuth and Elevation Estimation:

- (a) For each harmonic h_q, first select $\gamma_{i,j} \in \Gamma_{i,j}^{h_q}$ such that $|\gamma_{i,j} - \hat{\gamma}_{i,j}(\hat{\phi}_k, \hat{\theta}_k)|$ is minimum for each pair of microphones $i \neq j \in \{1, \cdots, 4\}$, and then use (9) to solve for U and convert U in to azimuth ϕ^{h_q} and elevation θ^{h_q} angles.

- (b) Select $\gamma_{i,j} \in \Gamma_{i,j}$ such that $|\gamma_{i,j} - \hat{\gamma}_{i,j}(\hat{\phi}_k, \hat{\theta}_k)|$ is minimum, where $\Gamma_{i,j} = \cup \left\{\Gamma_{i,j}^{h_q}\right\}, \ \forall q \in \{1, \cdots, m\}$. Use $\gamma_{i,j}$ in (9) to estimate U and convert U to get azimuth ϕ^H and elevation θ^H.

- (c) Compute new azimuth and elevation of the helicopter as

$$\begin{aligned} \phi_k &= \tfrac{1}{N} \textstyle\sum_{i=1}^{N} \Phi(i) \\ \theta_k &= \tfrac{1}{N} \textstyle\sum_{i=1}^{N} \Theta(i), \end{aligned} \tag{14}$$

  where $\Phi \subseteq \left\{\phi^{h_1}, \cdots, \phi^{h_m}, \phi^H\right\}$, such that $|\Phi(i) - \hat{\phi}_k| < 10, \ \forall i$, N is the number of elements in Φ, and $\Theta(i) - \hat{\theta}_k > 10$ if $\hat{\theta}_k < 80$ or $\Theta(i) - \hat{\theta}_k < 10$ if $\hat{\theta}_k \geq 80$.

Now the algorithm for estimation of the azimuth and elevation angles from the data will be presented:

Algorithm

- Step 1: Set $k = 1$. Get the k^{th} second data from all the microphones in a sensor array.
- Step 2: Generate FFT of the data and estimate the fundamental in $10 - 14$ Hz range and get the significant harmonics $H = \{h_1, \cdots, h_m\}$.
- Step 3: Estimate the azimuth angle of the target using all or a subset of microphone data using MVDR algorithm. This value is denoted by ϕ_{k1}.
- Step 4: If $k > 10$, predict the value of azimuth using the past 10 values. Denote the predicted value by ϕ_k. If $k \leq 10$, set $\phi_k = \phi_{k1}$.
- Step 5: Generate AoA tables $A_{i,j}(\phi_{k1}, \theta)$ and $A_{i,j}(\phi_k, \theta)$, for all $\theta \in \{1, \cdots, 90\}$ and $i \neq j \in \{1, 2, \cdots, n\}$, where n is the number of microphones in a sensor array.
- Step 6: If $k = 1$, set elevation $\theta_k = 1$; else use the estimated value of the elevation for $(k-1)^{th}$ time step. For i and j^{th} pair, compute the expected AoA as

  $$\hat{\gamma}_{i,j} = \frac{\hat{\gamma}_{i,j}(\phi_{k1}, \theta_k) + \hat{\gamma}_{i,j}(\phi_k, \theta_k)}{2}$$

  This average is used to overcome the variations in estimated value of azimuth from the data.
- Step 7: Estimate the new azimuth ϕ_k and elevation θ_k as described previously using (14).
- Step 8: Increment k and get the data. Go to Step 2.

In the next section we present the results.

4 Experimental Results

The algorithm presented in the previous section is applied on the data collected in the field. Figure 7 shows the location of two tetrahedral acoustic array sensors and the helicopter ground track. The helicopter was flown on a pre-determined path so that – variety of azmuth and elevation patterns could be obtained to test the tracking algorithm. Figure 10 shows the estimation of azimuth by the algorithm and the ground truth for site 1. Similarly, Figure 11 shows the estimation of the elevation angle of the helicopter by the algorithm and the ground truth. Figure 12 shows both azimuth and elevation estimations for site 2. From these figures, we find that the algorithm is able to estimate the azimuth and elevation reasonably well. Whenever the azimuth estimation is either wrong or failed to estimate (see 'gap in estimation' on Figure 10), the elevation is estimated wrongly. The gaps in estimation of azimuth occur whenever there is signal fading. The fading can be seen on the spectrogram of the data in Figure 9.

Figure 10: Algorithm results for azimuth

Figure 11: Algorithm results for elevation

5 Conclusion

In this paper we presented a new paradigm to estimate both azimuth and elevation angles using a single array. The results are presented. Using pairs of microphones provides a rich set of AoAs to predict the pointing vector U and, similarly, using several harmonics to estimate U and taking the average makes the estimation of elevation angles feasible, unlike the MVDR algorithm. Whenever there is fading in the signal, the azimuth estimation and elevation estimations are found to be poor. Future research will concentrate on improving the signal processing techniques to cover the fading signals.

Due to the refraction of the sound waves, the true elevation angles (geometrical angles) will be different from the actual elevation angles present at the sensor

Figure 12: Algorithm results for site 2

array. Some amount of correction needs to be made to the estimated values. The correction factor depends on the temperature profile of the atmosphere at the test site, and the altitude and distance of the helicopter from the sensor array. Since the azimuth angles are reasonably accurate, XY coordinates of the helicopter can be estimated using triangulation. The altitude of the helicopter will depend on the elevation estimations. Since estimated elevation angles are reasonable, the estimation of the altitude of the helicopter is also reasonable.

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